AMPLITUDE ANALYSIS OF REACTIONS $\pi^- p \to \eta\pi^- p$ AND $\pi^- p \to \eta\pi^0 n$ MEASURED ON POLARIZED TARGET AND THE EXOTIC 1^{-+} MESON.

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Abstract

Recently several experimental groups analysed data on $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ reactions with exotic 1^{-+} P-wave and found a conflicting evidence for an exotic meson $I = 11^{-+}(1405)$. High statistics data on these reactions are presently analysed by BNL E852 Collaboration. All these analyses are based on the crucial assumption that the production amplitudes do not depend on nucleon spin. This assumption is in sharp conflict with the results of measurements of $\pi^- p \to \pi^-\pi^+ n$, $\pi^+ n \to \pi^+\pi^- p$ and $K^+ n \to K^+\pi^- p$ on polarized targets at CERN which find a strong dependence of production amplitudes on nucleon spin. To ascertain the existence of exotic meson $1^{-+}(1405)$, it is necessary to perform a model-independent amplitude analysis of reactions $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$. We demonstrate that measurements of these reactions on transversely polarized targets enable the required model independent amplitude analysis without the assumption that produc-

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tion amplitudes are independent on nucleon spin. We suggest that high statistics measurements of reactions $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ be made on polarized targets at BNL and at Protvino IHEP, and that model-independent amplitude analyses of this polarized data be performed to advance hadron spectroscopy on the level of spin dependent production amplitudes.

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I. INTRODUCTION

Search for meson states with non-$q\overline{q}$ quantum numbers such as $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, \ldots$ has attracted much attention in recent years. Of special importance are reactions $\pi^- p \to \eta\pi^- p$, $\pi^- p \to \eta\pi^0 n$ and $\pi^- p \to \eta\eta' n$. In these reactions the dimeson system is produced predominantly in spin states $J = 0$ (S-wave), $J = 1$ (P-wave) and $J = 2$ (D-wave) for masses up 2.6 GeV. It is the P-wave which is of special interest as it carries exotic quantum numbers $I = 1 J^{PC} = 1^{-+}$ for reactions $\pi^- p \to \eta\pi^- p$, $\pi^- p \to \eta\pi^0 n$ and $I = 0 J^{PC} = 1^{-+}$ for $\pi^- p \to \eta\eta' n$.

Measurements of $\pi^- p \to \eta\pi^0 n$ at 100 GeV/c by the GAMS Collaboration [1] found large Forward-Backward asymmetry with pronounced features at around 1300 MeV. Similar Forward-Backward asymmetry was found in measurements of $\pi^- p \to \eta\pi^- p$ at 6.3 GeV/c by KEK E-179 Collaboration [2,3]. The higher statistics measurement of $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ reactions at 18 GeV/c by BNL E-852 Collaboration [4] confirmed significant Forward-Backward asymmetry in the data beginning at invariant mass of about 1.2 GeV in both reactions. The behaviour of the asymmetry suggests the presence of large exotic P-wave interfering with dominant D-wave with its $a_2(1320)$ resonance. The question arises whether there is a resonant production of the $\eta\pi^-$ or $\eta\pi^0$ state in the exotic P-wave. The reliable determination of existence of exotic resonance in 1^{-+} P-wave requires model-independent amplitude analysis of the data.

The reactions $\pi^- p \to \eta\pi^- p$, $\pi^- p \to \eta\pi^0 n$ and $\pi^- p \to \eta\eta' n$ are described by 14 spin dependent production amplitudes: 2 S-wave amplitudes S_n, 6 P-wave amplitudes P^0_n, P^-_n, P^+_n and 6 D-wave amplitudes D^0_n, D^-_n, D^+_n where $n = 0, 1$ is nucleon helicity flip $n = |\lambda_p - \lambda_n|$. The amplitudes S_n, P^0_n, D^0_n describe the production with dimeson helicity $\lambda = 0$ and correspond to unnatural exchange. The amplitudes P^-_n, D^-_n and P^+_n, D^+_n describe production with dimeson helicity ± 1 and correspond to unnatural and natural exchanges, respectively.

All previous amplitude analyses of reactions $\pi^- p \to \eta\pi^- p$, $\pi^- p \to \eta\pi^0 n$ and $\pi^- p \to \eta\eta' n$ on unpolarized targets are model dependent. They use a very strong simplifying assumption

that the production amplitudes do not depend on nucleon spin [5–7]. The purpose of this assumption is to reduce the number of amplitudes by half and thus to enable the amplitude analysis of unpolarized moments measured in these reactions to proceed. These analyses simply ignore the nucleon helicity flip index n.

Using such enabling assumption, the different collaborations found the exotic $I = 11^{-+}$ meson but in different amplitudes. The GAMS Collaboration reported $1^{-+}(1405)$ state with a width of 180 MeV [1] observed only in the amplitude $|P^0|^2$. The KEK E-179 Collaboration [2,3] found $|P^-|^2$ nonresonating but found resonance $1^{-+}(1323)$ with a width of 143 MeV in the amplitude $|P^+|^2$ and possibly in $|P^0|^2$. The VES Collaboration [8] measured $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta'\pi^- p$ at 37 GeV/c at IHEP Protvino, and found possible $1^{-+}(1400)$ state only in the amplitude $|P^+|^2$. Amplitude analysis of BNL E-852 Collaboration higher statistics data at 18 GeV/c is in progress, but it also uses the simplifying assumption that production amplitudes do not depend on nucleon spin. All these analyses are subjected to an eight-fold ambiguity and in Ref. 2, 3 and 8 all eight solutions are presented.

For completeness we note that GAMS Collaboration measured reaction $\pi^- p \to \eta\eta' n$ at 38 GeV/c [9] and found evidence for a new state $X(1920)$. The unusual production and decay properties could be understood if $X(1920)$ had a non-$q\overline{q}$ structure, being either a 0^{++} or 2^{++} glueball or $I = 01^{-+}$ exotic meson. Unfortunately, the low statistics does not allow even a model dependent amplitude analysis.

The simplifying assumption that the production amplitudes do not depend on nucleon spin is not necessary in measurements on polarized targets. In 1978, Lutz and Rybicki showed [10] that measurements of reactions $\pi N \to \pi^+\pi^- N$ and $KN \to K\pi N$ on polarized target yield enough observables that model independent amplitude analysis is possible determining the spin dependent production amplitudes. The measurement of these reactions is of special interest to hadron spectroscopy because they permit to study the spin dependence of resonance production directly on the level of spin-dependent production amplitudes. Several such measurements were done at CERN-PS.

The high statistics measurement of $\pi^- p \to \pi^-\pi^+ n$ at 17.2 GeV/c on unpolarized target

[11] was later repeated with a transversely polarized target at the same energy [12–17]. Model independent amplitude analyses were performed for various intervals of dimeson mass at small momentum transfers $-t = 0.005 - 0.2$ (GeV/c)^2 [12–15], and over a large interval of momentum transfer $-t = 0.2 - 1.0$ (GeV/c)^2 [16,17].

Additional information was provided by the first measurement of $\pi^+ n \to \pi^+\pi^- p$ and $K^+ n \to K^+\pi^- p$ reactions on polarized deuteron target at 5.98 and 11.85 GeV/c [18,19]. The data allowed to study the t-evolution of mass dependence of moduli of amplitudes [20]. Detailed amplitude analyses [21,22] determined the mass dependence of amplitudes at larger momentum transfers $-t = 0.2 - 0.4$ (GeV/c)^2.

The crucial finding of all these measurements was the evidence for strong dependence of production amplitudes on nucleon spin. The process of resonance production is very closely related to nucleon transversity, or nucleon spin component in direction perpendicular to the production plane. For instance, in $\pi^- p \to \pi^-\pi^+ n$ at small t and dipion masses below 1000 MeV, all amplitudes with recoil nucleon transversity "down" are smaller than transversity "up" amplitudes, irrespective of dimeson spin and helicity. In particular, the S-wave amplitude with recoil nucleon transversity "up" is found to resonate at 750 MeV in both solutions [23–25] irrespective of the method of amplitude analysis [25], while the S-wave amplitude with recoil nucleon transversity "down" is nonresonating and large in both solutions. It is important to stress that the discovery of the narrow scalar state $\sigma(750)$ in $\pi^- p \to \pi^-\pi^+ n$ and $\pi^+ n \to \pi^+\pi^- p$ [24,25] was possible only because these reactions were measured on polarized targets which allowed the model-independent determination of the spin dependent production amplitudes.

The assumption that production amplitudes in $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ do not depend on nucleon spin contradicts all that we have learned from the measurements of $\pi N \to \pi^+\pi^- N$ on polarized targets at CERN. Applied to reactions $\pi^- p \to \pi^-\pi^+ n$ and $\pi^+ n \to \pi^+\pi^- p$, the assumption has observable consequences that can be tested directly in measurements on polarized targets. In the previous paper [26] we have shown how all these consequences are in contradiction with CERN polarized data on $\pi N_\uparrow \to \pi^+\pi^- N$ and

$K^+ n_\uparrow \to K^+ \pi^- p$ (see Fig. 1 and 2 of Ref. 26). We must conclude that the CERN polarized data invalidate the assumption that production amplitudes do not depend on nucleon spin. Consequently, some of the results of analyses of $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ may not be reliable.

The question of reliability of amplitude analyses based on assumption of independence of production amplitudes on nucleon spin is of special importance to searches for exotic resonances like $1^{-+}(1405)$ in $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ reactions, or confirmation of the narrow $\sigma(750)$ state in $\pi^- p \to \pi^0\pi^0 n$ reaction.

Only a model independent analysis will resolve questions concerning the existence of such resonances which are not seen in the integrated mass spectrum but only on the level of spin dependent production amplitudes.

In the previous paper [26] we have shown how measurements of $\pi^- p \to \pi^0\pi^0 n$ on polarized targets allow a model independent amplitude analysis of this reaction (and $\pi^- p \to \eta\eta n$). Using the results of Lutz and Rybicki [10], we show in this work that measurements of $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ on polarized target again allow a model independent determination of moduli of all production amplitudes and cosines of certain independent relative phases. We find an eight-fold ambiguity, which is the same situation as in model dependent analyses of unpolarized data.

We propose that high statistics measurements of $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ be made at Brookhaven Multiparticle Spectrometer and at IHEP Protvino in conjunction with measurements of $\pi^- p \to \pi^0\pi^0$ reaction on polarized target.

The paper is organized as follows. In Section II we review our basic notation and definitions of observables and amplitudes. In Section III we present the expressions for unpolarized and polarized moments in terms of amplitudes. In Section IV we discuss the method of model-independent amplitude analysis of data on $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ on polarized target. The paper closes with Section V where we present summary and our proposals.

II. BASIC FORMALISM.

The kinematical variables which describe the reactions $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ on a polarized proton target at rest are $(s, t, m, \theta, \phi, \psi, \delta)$ where s is the c.m.s. energy squared, t is four-momentum transfer to the nucleon squared, and m is the invariant mass of the $\eta\pi$ system. The angles θ, ϕ describe the direction of η in the $\eta\pi^-$ or $\eta\pi^0$ rest frame. The angle ψ is the angle between the direction of target transverse polarization and the normal $\vec{n}$ to the scattering plane (Fig. 1). The direction of normal $\vec{n}$ is defined according to Basel convention by $\vec{p}_\pi \times \vec{p}_{\eta\pi}$ where $\vec{p}_\pi$ and $\vec{p}_{\eta\pi}$ are the incident and dimeson momenta in the target proton rest frame. The angle δ is the angle between the direction of target polarization vector and its transverse component (Fig. 1). The analysis is usually carried out in the t-channel helicity frame for the $\eta\pi$ dimeson system. The helicities of initial and final nucleons are always defined in the s-channel helicity frame.

When the polarization of the recoil nucleon is not measured, the unnormalized angular distribution of $\eta\pi^-$ or $\eta\pi^0$ production on polarized protons at rest at fixed s, m and t can be written [10] as

$$I(\Omega, \psi, \delta) = I_U(\Omega) + P_T \cos\psi I_C(\Omega) + P_T \sin\psi I_S(\Omega) + P_L I_L(\Omega) \tag{2.1}$$

where $P_T = P\cos\delta$ and $P_L = P\sin\delta$ are the transverse and longitudinal components of target polarization $\vec{P}$ with respect to the incident momentum (Fig. 1). In the data analysis of angular distribution of the dimeson system, it is convenient to use expansions of the angular distributions in terms of spherical harmonics. In the notation of Lutz and Rybicki [10] we have

$$I_U(\Omega) = \sum_{L,M} t^L_M \mathrm{Re}Y^L_M(\Omega) \tag{2.2}$$

$$I_C(\Omega) = \sum_{L,M} p^L_M \mathrm{Re}Y^L_M(\Omega)$$

$$I_S(\Omega) = \sum_{L,M} r^L_M \mathrm{Im}Y^L_M(\Omega)$$

$$I_L(\Omega) = \sum_{L,M} q_M^L \mathrm{Im} Y_M^L(\Omega)$$

The moments t_M^L are unpolarized and are measured in experiments on unpolarized targets. Experiments with transversely polarized targets measure transverse polarized moments p_M^L and r_M^L but not the longitudinal polarized moments q_M^L. More details on these observables are given in Ref. 10 and 26.

The reaction $\pi^- p \to \eta\pi^- p$ (or $\pi^- p \to \eta\pi^0 n$) is described by production amplitude $H_{\lambda_n,0\lambda_p}(s,t,m,\theta,\phi)$ where λ_p and λ_n are the helicities of the proton and neutron, respectively. The production amplitudes can be expressed in terms of production amplitudes corresponding to definite dimeson spin J and helicity λ using an angular expansion

$$H_{\lambda_n,0\lambda_p} = \sum_{J=0}^{\infty} \sum_{\lambda=-J}^{+J} (2J+1)^{1/2} H^J_{\lambda\lambda_n,0\lambda_p}(s,t,m) d^J_{\lambda 0}(\theta) e^{i\lambda\phi} \tag{2.3}$$

In the following we will consider only S-wave ($J=0$), P-wave ($J=1$) and D-wave ($J=2$) amplitudes. Since the experimental moments with $M > 2$ vanish, we will restrict the dimeson helicity λ only to values $\lambda = 0$ and $\lambda = \pm 1$.

The amplitudes $H^J_{\lambda\lambda_n,0\lambda_p}(s,t,m)$ can be expressed in terms of nucleon helicity amplitudes with definite t-channel exchange naturality. The nucleon s-channel helicity amplitudes describing the production of $\eta\pi^-$ (or $\eta\pi^0$) system in the S-, P- and D-wave states are:

$$0^-\frac{1}{2}^+ \to 0^+\frac{1}{2}^+ \;:\; H^0_{0+,0+} = S_0,\; H^0_{0+,0-} = S_1 \tag{2.4}$$

$$0^-\frac{1}{2}^+ \to 1^-\frac{1}{2}^+ \;:\; H^1_{0+,0+} = P^0_0,\; H^1_{0+,0-} = P^0_1$$

$$H^1_{\pm 1+,0+} = \frac{P_0^+ \pm P_0^-}{\sqrt{2}},\; H^1_{\pm 1+,0-} = \frac{P_1^+ \pm P_1^-}{\sqrt{2}}$$

$$0^-\frac{1}{2}^+ \to 2^+\frac{1}{2}^+ \;:\; H^2_{0+,0+} = D^0_0,\; H^2_{0+,0-} = D^0_1$$

$$H^2_{\pm 1+,0+} = \frac{D_0^+ \pm D_0^-}{\sqrt{2}},\; H^2_{\pm 1+,0-} = \frac{D_1^+ \pm D_1^+}{\sqrt{2}}$$

At large s, the amplitudes $S_n, P^0_n, P^-_n, D^0_n, D^-_n, n = 0, 1$ are dominated by the unnatural exchanges. The amplitudes $P^+_n, D^+_n, n = 0, 1$ are dominated by natural exchanges. The index $n = |\lambda_p - \lambda_n|$ is nucleon helicity flip.

The observables measured in experiments on transversely polarized targets are most simply related to nucleon transversity amplitudes of definite naturality [10,19,27]. With $k = 1/\sqrt{2}$, they are defined as follows:

$$S = k(S_0 + iS_1)\ ,\ \overline{S} = k(S_0 - iS_1) \tag{2.5}$$

$$P^0 = k(P^0_0 + iP^0_1)\ ,\ \overline{P}^0 = k(P^0_0 - iP^0_1)$$

$$P^- = k(P^-_0 + iP^-_1)\ ,\ \overline{P}^- = k(P^-_0 - iP^-_1)$$

$$P^+ = k(P^+_0 - iP^+_1)\ ,\ \overline{P}^+ = k(P^+_0 + iP^+_1)$$

$$D^0 = k(D^0_0 + iD^0_1)\ ,\ \overline{D}^0 = k(D^0_0 - iD^0_1)$$

$$D^- = k(D^-_0 + iD^-_1)\ ,\ \overline{D}^- = k(D^-_0 - iD^-_1)$$

$$D^+ = k(D^+_0 - iD^+_1)\ ,\ \overline{D}^+ = k(D^+_0 + iD^+_1)$$

The nucleon helicity and nucleon transversity amplitudes differ in the quantization axis for the nucleon spin. The transversity amplitudes $S, P^0, P^-, P^+, D^0, D^-, D^+$ $(\overline{S}, \overline{P}^0, \overline{P}^-, \overline{P}^+, \overline{D}^0, \overline{D}^-, \overline{D}^+)$ describe the production of $\eta\pi$ state with the recoil nucleon spin antiparallel or down (parallel or up) relative to the normal $\vec{n}$ to the production plane.

III. OBSERVABLES IN TERMS OF AMPLITUDES.

It is useful to express the moments t^L_M and p^L_M in terms of quantities that do not depend explicitly on whether we use nucleon helicity or nucleon transversity amplitudes. The required quantities are spin-averaged partial wave intensity

$$I_A = |A|^2 + |\overline{A}|^2 = |A_0|^2 + |A_1|^2 \tag{3.1}$$

and partial wave polarization

$$P_A = |A|^2 - |\overline{A}|^2 = 2\epsilon_A \mathrm{Im}(A_0 A_1^*) \tag{3.2}$$

where $\epsilon_A = +1$ for $A = S, P^0, P^-, D^0, D^-$ and $\epsilon_A = -1$ for $A = P^+, D^+$. We also need spin-averaged interference terms

$$R(AB) = \mathrm{Re}(AB^* + \overline{A}\,\overline{B}^*) = \mathrm{Re}(A_0 B_0 + \epsilon_A \epsilon_B A_1 B_1) \tag{3.3}$$

$$Q(AB) = \mathrm{Re}(AB^* - \overline{A}\,\overline{B}^*) = \mathrm{Re}(\epsilon_B A_0 B_1^* - \epsilon_A A_1 B_0^*) \tag{3.4}$$

Then moments t^L_M are expressed in terms of intensities I_A and interference terms $R(AB)$. The moments p^L_M are expressed in terms of polarizations P_A and interference terms $Q(AB)$. The moments r^L_M are interferences between the natural and unnatural exchange amplitudes. To describe moments r^L_M, it is useful to introduce notation

$$N(AP^+) = \mathrm{Re}(AP^{+*} - \overline{A}\,\overline{P}^{+*}) \tag{3.5}$$

$$N(AD^+) = \mathrm{Re}(AD^{+*} - \overline{A}\,\overline{D}^{+*})$$

where $A = S, P^0, P^-, D^0, D^-$.

Using the results of the Lutz and Rybicki [10] we obtain the following expressions for moments with $c = \sqrt{4\pi}$:

Unpolarized moments t^L_M (3.6)

$$ct^0_0 = I_S + I_{P^0} + I_{P^-} + I_{P^+} + I_{D^0} + I_{D^-} + I_{D^+}$$

$$ct^1_0 = 2R(SP^0) + \frac{4}{\sqrt{5}}R(P^0D^0) + 2\sqrt{\frac{3}{5}}[R(P^-D^-) + R(P^+D^+)]$$

$$ct^1_1 = 2\sqrt{2}R(SP^-) + 2\sqrt{\frac{6}{5}}R(P^0D^-) - 2\sqrt{\frac{2}{5}}R(P^-D^0)$$

$$ct_0^2 = \frac{2}{\sqrt{5}}I_{P^0} - \frac{1}{\sqrt{5}}(I_{P^-} + I_{P^+}) + 2R(SD^0) + \frac{2}{7}\sqrt{5}I_{D^0} + \frac{\sqrt{5}}{7}(I_{D^-} + I_{D^+})$$

$$ct_1^2 = 2\sqrt{\frac{6}{5}}R(P^0P^-) + 2\sqrt{2}R(SD^-) + \frac{2\sqrt{10}}{7}R(D^0D^-)$$

$$ct_2^2 = \sqrt{\frac{6}{5}}(I_{P^-} - I_{P^+}) + \frac{\sqrt{30}}{7}(I_{D^-} - I_{D^+})$$

$$ct_0^3 = 6\sqrt{\frac{3}{35}}R(P^0D^0) - \frac{6}{\sqrt{35}}[R(P^-D^-) + R(P^+D^+)]$$

$$ct_1^3 = 8\sqrt{\frac{3}{35}}R(P^0D^-) + \frac{12}{\sqrt{35}}R(P^-D^0)$$

$$ct_2^3 = 2\sqrt{\frac{6}{7}}[R(P^-D^-) - R(P^+D^+)]$$

$$ct_0^4 = \frac{6}{7}I_{D^0} - \frac{4}{7}(I_{D^-} + I_{D^+})$$

$$ct_1^4 = \frac{4}{7}\sqrt{15}R(D^0D^-)$$

$$ct_2^4 = \frac{2\sqrt{10}}{7}(I_{D^-} - I_{D^+})$$

Polarized moments p_M^L (3.7)

$$cp_0^0 = P_S + P_{P^0} + P_{P^-} - P_{P^+} + P_{D^0} + P_{D^-} - P_{D^+}$$

$$cp_0^1 = 2Q(SP^0) + \frac{4}{\sqrt{5}}Q(P^0D^0) + 2\sqrt{\frac{3}{5}}[Q(P^-D^-) - Q(P^+D^+)]$$

$$cp_1^1 = 2\sqrt{2}Q(SP^-) + 2\sqrt{\frac{6}{5}}Q(P^0D^-) - 2\sqrt{\frac{2}{5}}Q(P^-D^0)$$

$$cp_0^2 = \frac{2}{\sqrt{5}}P_{P^0} - \frac{1}{\sqrt{5}}(P_{P^-} - P_{P^+}) + 2Q(SD^0) + \frac{2\sqrt{5}}{7}P_{D^0} + \frac{\sqrt{5}}{7}(P_{D^-} - P_{D^+})$$

$$cp_1^2 = 2\sqrt{\frac{6}{5}}Q(P^0P^-) + 2\sqrt{2}Q(SD^-) + \frac{2\sqrt{10}}{7}Q(D^0D^-)$$

$$cp_2^2 = \sqrt{\frac{6}{5}}(P_{P^-} + P_{P^+}) + \frac{\sqrt{30}}{7}(P_{D^-} + P_{D^+})$$

$$cp_0^3 = 6\sqrt{\frac{3}{35}}Q(P^0D^0) - \frac{6}{\sqrt{35}}[Q(P^-D^-) - Q(P^+D^+)]$$

$$cp_1^3 = 8\sqrt{\frac{3}{35}}Q(P^0D^-) + \frac{12}{\sqrt{35}}Q(P^-D^0)$$

$$cp_2^3 = 2\sqrt{\frac{6}{7}}[Q(P^-D^-) + Q(P^+D^+)]$$

$$cp_0^4 = \frac{6}{7}P_{D^0} - \frac{4}{7}(P_{D^-} - P_{D^+})$$

$$cp_0^4 = \frac{4}{7}\sqrt{15}Q(D^0D^-)$$

$$cp_2^4 = \frac{2\sqrt{10}}{7}(P_{D^-} + P_{D^+})$$

Polarized moments r_M^L (3.8)

$$cr_1^1 = -2\sqrt{2}N(SP^+) - 2\sqrt{\frac{2}{5}}N(D^0P^+) - 2\sqrt{\frac{6}{5}}N(P^0D^+)$$

$$cr_1^2 = -2\sqrt{\frac{6}{5}}N(P^0P^+) - 2\sqrt{2}N(SD^+) - \frac{2\sqrt{10}}{7}N(D^0D^+)$$

$$cr_2^2 = -2\sqrt{\frac{6}{5}}N(P^-P^+) - \frac{2\sqrt{30}}{7}N(D^-D^+)$$

$$cr_1^3 = +\frac{12}{\sqrt{35}}N(D^0P^+) - 8\sqrt{\frac{3}{35}}N(P^0D^+)$$

$$cr_2^3 = -2\sqrt{\frac{6}{7}}N(D^-P^+) - 2\sqrt{\frac{6}{7}}N(P^-D^+)$$

$$cr_1^4 = -\frac{4}{7}\sqrt{15}N(D^0D^+)$$

$$cr_2^4 = -\frac{4}{7}\sqrt{10}N(D^-D^+)$$

IV. MODEL INDEPENDENT AMPLITUDE ANALYSIS.

Our starting point is the observation of symmetry in the relations for moments t_M^L and p_M^L. We find that we get p_M^L from t_M^L by replacing intensities I_A by polarizations $\epsilon_A P_A$, $\epsilon = +1$ for $A = S, P^0, P^-, D^0, D^-$ and $\epsilon_A = -1$ for $A = P^+, D^+$, and by replacing the interference terms $R(AB) \to Q(AB)$ for unnatural exchange amplitudes and $R(P^+D^+) \to -Q(P^+D^+)$ for natural exchange amplitudes. To solve the system of equations t_M^L and p_M^L it will be useful to work with transversity amplitudes. Then the definitions (3.1)–(3.4) suggest to construct two sets of equations corresponding to the sum and difference of the moments t_M^L and p_M^L. In this way we get two independent sets of equations for amplitudes of opposite transversity.

The first set of new observables reads:

$$a_1 = \frac{c}{2}(t_0^0 + p_0^0) = |S|^2 + |P^0|^2 + |P^-|^2 + |\overline{P}^+|^2 + |D^0|^2 + |D^-|^2 + |\overline{D}^+|^2 \tag{4.1}$$

$$a_2 = \frac{c}{2}(t_0^1 + p_0^1) = 2\mathrm{Re}(SP^{0*}) + \frac{4}{\sqrt{5}}\mathrm{Re}(P^0D^{0*}) + 2\sqrt{\frac{3}{5}}[\mathrm{Re}(P^-D^{-*}) + \mathrm{Re}(\overline{P}^+\overline{D}^{+*})]$$

$$a_3 = \frac{c}{2}(t_1^1 + p_1^1) = 2\sqrt{2}\mathrm{Re}(SP^{-*}) + 2\sqrt{\frac{6}{5}}\mathrm{Re}(P^0D^{-*}) - 2\sqrt{\frac{2}{5}}\mathrm{Re}(P^-D^{0*})$$

$$a_4 = \frac{c}{2}(t_0^2+p_0^2) = \frac{2}{\sqrt{5}}|P^0|^2 - \frac{1}{\sqrt{5}}(|P^-|^2+|\overline{P}^+|^2) + 2\mathrm{Re}(SD^{0*}) + \frac{2\sqrt{5}}{7}|D^0|^2 + \frac{\sqrt{5}}{7}(|D^-|^2+|\overline{D}^+|^2)$$

$$a_5 = \frac{c}{2}(t_1^2+p_1^2) = 2\sqrt{\frac{6}{5}}\mathrm{Re}(P^0P^{-*}) + 2\sqrt{2}\mathrm{Re}(SD^{-*}) + \frac{2\sqrt{10}}{7}\mathrm{Re}(D^0D^{-*})$$

$$a_6 = \frac{c}{2}(t_2^2+p_2^2) = \sqrt{\frac{6}{5}}(|P^-|^2 - |\overline{P}^+|^2) + \frac{\sqrt{30}}{7}(|D^-|^2 - |\overline{D}^+|^2)$$

$$a_7 = \frac{c}{2}(t_0^3+p_0^3) = 6\sqrt{\frac{3}{35}}\mathrm{Re}(P^0D^{0*}) - \frac{6}{\sqrt{35}}[\mathrm{Re}(P^-D^{-*}) + \mathrm{Re}(\overline{P}^+\overline{D}^{+*})]$$

$$a_8 = \frac{c}{2}(t_1^3+p_1^3) = 8\sqrt{\frac{3}{35}}\mathrm{Re}(P^0D^{-*}) + \frac{12}{\sqrt{35}}\mathrm{Re}(P^-D^{0*})$$

$$a_9 = \frac{c}{2}(t_2^3+p_2^3) = 2\sqrt{\frac{6}{7}}[\mathrm{Re}(P^-D^{-*}) - \mathrm{Re}(\overline{P}^+\overline{D}^{+*})]$$

$$a_{10} = \frac{c}{2}(t_0^4+p_0^4) = \frac{6}{7}|D^0|^2 - \frac{4}{7}(|D^-|^2 + |\overline{D}^+|^2)$$

$$a_{11} = \frac{c}{2}(t_1^4+p_1^4) = \frac{4}{7}\sqrt{15}\mathrm{Re}(D^0D^{-*})$$

$$a_{12} = \frac{c}{2}(t_2^4+p_2^4) = \frac{2\sqrt{10}}{7}(|D^-|^2 - |\overline{D}^+|^2)$$

The first set of equations (4.1) involves 7 moduli

$$|S|, |P^0|, |P^-|, |\overline{P}^+|, |D^0|, |D^-|, |\overline{D}^+| \tag{4.2}$$

and 10 cosines of relative phases between unnatural exchange amplitudes

$$\cos(\gamma_{SP^0}), \cos(\gamma_{SP^-}), \cos(\gamma_{SD^0}), \cos(\gamma_{SD^-}) \tag{4.3}$$

$$\cos(\gamma_{P^0P^-}), \cos(\gamma_{P^0D^0}), \cos(\gamma_{P^0D^-}) \tag{4.4}$$

$$\cos(\gamma_{P^-D^0}), \cos(\gamma_{P^-D^-}), \cos(\gamma_{D^0D^-}) \tag{4.5}$$

and 1 cosine of relative phase between natural exchange amplitudes

$$\cos(\overline{\gamma}_{P^+D^+}) \tag{4.6}$$

The second set of observables $\overline{a}_i, i = 1, 2, \ldots, 12$ corresponding to the differences of moments t^L_M and p^L_M involves the same moduli and cosines as the first set but for amplitudes of opposite transversity:

7 moduli

$$|\overline{S}|, |\overline{P}^0|, |\overline{P}^-|, |P^+|, |\overline{D}^0|, |\overline{D}^-|, |D^+| \tag{4.7}$$

10 cosines of relative phases between unnatural exchange amplitudes

$$\cos(\overline{\gamma}_{SP^0}), \cos(\overline{\gamma}_{SP^-}), \cos(\overline{\gamma}_{SD^0}), \cos(\overline{\gamma}_{SD^-}) \tag{4.8}$$

$$\cos(\overline{\gamma}_{P^0P^-}), \cos(\overline{\gamma}_{P^0D^0}), \cos(\overline{\gamma}_{P^0D^-})$$

$$\cos(\overline{\gamma}_{P^-D^0}), \cos(\overline{\gamma}_{P^-D^-}), \cos(\overline{\gamma}_{D^0D^-}) \tag{4.9}$$

1 cosine of relative phase between natural exchange amplitudes

$$\cos(\gamma_{P^+D^+}) \tag{4.10}$$

We will now show that the cosines (4.4) and (4.5) can be expressed in terms of the cosines (4.3). For instance, we can write

$$\gamma_{P^0P^-} = \phi_{P^0} - \phi_{P^-} = (\phi_S - \phi_{P^-}) - (\phi_S - \phi_{P^0}) = \gamma_{SP^-} - \gamma_{SP^0} \tag{4.11}$$

Then

$$\cos(\gamma_{P^0P^-}) = \cos(\gamma_{SP^0})\cos(\gamma_{SP^-}) + \sin(\gamma_{SP^0})\sin(\gamma_{SP^-}) \tag{4.12}$$

Since the signs of the sines $\sin(\gamma_{SP^0})$ and $\sin(\gamma_{SP^-})$ are not known, we write

$$\sin(\gamma_{SP^0}) = \epsilon_{SP^0}|\sin(\gamma_{SP^0})| \tag{4.13}$$

$$\sin(\gamma_{SP^-}) = \epsilon_{SP^-}|\sin(\gamma_{SP^-})|$$

Hence

$$\cos(\gamma_{P^0P^-}) = \cos(\gamma_{SP^0})\cos(\gamma_{SP^-}) + \epsilon_{P^0P^-}\sqrt{(1-\cos^2\gamma_{SP^0})(1-\cos^2\gamma_{SP^0})} \tag{4.14}$$

where $\epsilon_{P^0P^-} = \pm 1$ is the sign ambiguity. The remaining cosines in (4.4) and (4.5) can be written in the form similar to (4.14) with their own sign ambiguities. The sign ambiguities of cosines (4.4) and (4.5) can be expressed in terms of sign ambiguities corresponding to the sines $\sin(\gamma_{SP^0})$, $\sin(\gamma_{SP^-})$, $\sin(\gamma_{SD^0})$ and $\sin(\gamma_{SD^-})$. We can write

$$\epsilon_{P^0P^-} = \epsilon_{SP^0}\epsilon_{SP^-} \tag{4.15}$$

$$\epsilon_{P^0D^0} = \epsilon_{SP^0}\epsilon_{SD^0}$$

$$\epsilon_{P^0D^-} = \epsilon_{SP^0}\epsilon_{SD^-}$$

$$\epsilon_{P^-D^0} = \epsilon_{SP^-}\epsilon_{SD^0} \tag{4.16}$$

$$\epsilon_{P^-D^-} = \epsilon_{SP^-}\epsilon_{SD^-}$$

$$\epsilon_{D^0D^-} = \epsilon_{SD^0}\epsilon_{SD^-}$$

The reversal of all signs ϵ_{SP^0}, ϵ_{SP^-}, ϵ_{SD^0} and ϵ_{SD^-} yields the same signs in (4.15) and (4.16). The sign ambiguities (4.16) are not independent. They are uniquely determined by the sign ambiguities (4.15). Only sign ambiguities (4.15) are independent and there is 8 sign combinations in (4.15). The following table lists all eight allowed sets of sign ambiguities of cosines (4.4) and (4.5):

	1	2	3	4	5	5	7	8
$\epsilon_{P^0P^-}$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$-$
$\epsilon_{P^0D^0}$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$-$
$\epsilon_{P^0D^-}$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$
$\epsilon_{P^-D^0}$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
$\epsilon_{P^-D^-}$	$+$	$-$	$+$	$-$	$-$	$+$	$-$	$+$
$\epsilon_{D^0D^-}$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$

Using expressions like (4.14) for cosines (4.4) and (4.5), the number of unknowns is reduced to 12. With each choice of sign ambiguity from the above Table we have a set of 12 equations for 12 unknown which can be solved numerically or by χ^2 method. Of course, there is an eight-fold ambiguity and we obtain 8 solutions for moduli (4.2) and cosines (4.3) and (4.6) in each (m,t) bin. Since each solution is uniquely labeled by the choice of sign ambiguities, there is no problem linking solutions in neighbouring (m,t) bins. Similarly we obtain 8 solutions for moduli (4.7), cosines (4.8) and (4.11) from the second set of equations, $\overline{a}_i, i = 1,2,\ldots,12$.

The 8 solutions from the first set of equations $a_i, i = 1,2,\ldots,12$ are independent from the 8 solutions obtained from the second set of equations $\overline{a}_i, i = 1,2,\ldots,12$. Consequently, there will be a 64-fold ambiguity in the partial wave intensities which we can write

$$I_A(i,j) = |A(i)|^2 + |\overline{A}(j)|^2, i,j = 1,2,\ldots 8 \tag{4.17}$$

where $A = S, P^0, P^-, P^+, D^0, D^-, D^+$.

As in the case of amplitude analysis of $\pi^- p_\uparrow \to \pi^+\pi^+ n$ at 17.2 GeV/c [12–17], the unpolarized moments t^L_M should come from measurements on unpolarized targets.

V. SUMMARY.

Measurements of $\pi^- p \to \pi^-\pi^+ n$, $\pi^+ n \to \pi^+\pi^- p$ and $K^+ n \to K^+\pi^- p$ on polarized targets at CERN found evidence for a strong dependence of pion production amplitudes on

nucleon spin. This evidence invalidates the assumption [5,6] that production amplitudes in $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ reactions do not depend on nucleon spin. The amplitude analyses of these reactions based on the assumption of independence of production amplitudes on nucleon spin are thus insufficient and are likely to be unreliable. To ascertain the existence of exotic resonance $1^{-+}(1405)$ and study its properties, a reliable, model independent amplitude analysis is required. Nucleon spin is not only relevant to the dynamics of production processes. It also allows the model independent determination of spin-dependent production amplitudes from measurements of $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ on polarized targets, as we have shown. Our major assumption was that moments with $M > 2$ do not contribute to the angular distributions. This may not be true at large momentum transfers. In this case one has to use the formalism developed by I. Sakrejda [16] which takes into account the helicities $\lambda = \pm 2$ of the D-wave.

Instruments shape research and determine which discoveries are made. Polarized targets have proven themselves to be valuable and important tools of discovery. We propose that high statistics measurements of reactions $\pi^- p \to \eta\pi^- p$ and $\pi^- p \to \eta\pi^0 n$ be made on polarized targets at BNL Multiparticle Spectrometer and at IHEP in Protvino, in conjunction with high statistics measurements of $\pi^- p \to \pi^0\pi^0$ on polarized targets. Such experiments will be also feasible at the recently proposed Japanese Hadron Project (JHP). When built, JHP will be a high-intensity 50 GeV proton accelerator complex with high quality pion, kaon and antiproton secondary beams [28]. The availability of such secondary beams will make JHP an ideal facility for hadron spectroscopy using polarized targets in a search for new resonant states at the level of spin-dependent production amplitudes.

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FIGURES

FIG. 1. Definition of the coordinate system used to describe the target polarization $\vec{P}$ and the decay of the dimeson $\eta\pi^-$ system.